CONCERNING SELF-EXCITED OSCILLATIONS IN FRONTAL COMBUSTION OF A FUEL MIXTURE IN A RESONATOR WITH LUMPED PARAMETERS

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The conditions needed to stimulate harmonic self-excited oscillations in frontal combustion of a fuel mixture in the throat or cavity of a Helmhholtz resonator are considered. It is shown that they depend on the properties of the fuel mixture and the position and orientation of the front of the flame and can be satisfied only in the presence in the zone of combustion of a surface of discontinuity of the parameters that determine the rate of combustion. It is noted that the results of analysis do not come into conflict with the facts and dependences that are observed in combustion of fuel mixtures in tunnel burners, piston carburetors, and rocket engines.

In some technical devices, in the ignition of fuel mixtures, the process of combustion sometimes becomes unstable and harmonic pulsations of pressure appear arbitrarily in the combustion chamber. This disturbs the normal operation of the device and usually leads to its mechanical and thermal destruction. Instability develops without external periodic effects; more often than not it is induced by small fluctuational perturbations of pressure. The amplitude of fluctuations increases rapidly and attains limiting values. The range of frequencies of the fluctuations is large and depends on the geometry of the device.

These features of the process indicate that it has a self-oscillating character and depends on some internal properties of the system that ensure the compensation of the natural losses of vibrational energy in it. More than ten hypothetical mechanisms suggested as a result of long investigations and intended to determine the essence of the phenomenon have not resolved all of the problems, since thermoacoustic self-oscillations continue to be virtually uncontrolled even today. The problems of investigation of their nature, the development of means to control them, and their prevention continue to be of current interest.

Taken alone, the phenomenon considered is not unique. Conceptually similar self-oscillations are observed in electronic devices and control systems, where they not only have been long ago well studied and described, but have also been used widely with necessary controls. The experience of their investigations [1] shows that a typical self-oscillating system must contain an interacting:

1) energy source;
2) frequency-selecting element (usually, a resonator);
3) amplifier of perturbations with positive feedback.

The available descriptions of self-oscillating systems with combustion and the experience of the work with them do not give grounds to assume that they possess any fatal properties that, when being analyzed, exclude the possibility of using the experience acquired, for example, in investigations of self-excited electronic oscillators. Furthermore, this possibility is provided directly by the theory of dynamic analogies [2], which is based on the analogy of the functions of the elements of technical devices and unity of equations that describe electrical and mechanical systems. We shall take advantage of this possibility by isolating the above-noted typical components of master oscillators in systems with combustion and consider these components in their classical relationship. We shall do this for the case of low-frequency thermoacoustic self-oscillations, when the wavelength corresponding to this frequency is many times larger than the geometric dimensions of the device in which these oscillations appear.

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The problem of the energy source is simple enough. In the devices considered this is virtually only a burning fuel mixture. In all of the cases the energy of the translational motion of fuel [3] is either negligibly small in comparison with the energy of the heat source or is generally equal to zero and therefore it will not be taken into account.

At a low (as adopted above) frequency of vibrations, usually there is no great difficulty in isolating the rigid and inertial discrete elements in a device and thus to represent it as a resonator with lumped parameters. In this case, in the analysis of the conditions for the appearance of vibrations it is convenient to select, as a computational model, a Helmholtz resonator which "... in its classical form represents a vessel with a throat. On excitation of the resonator by a sound wave the air in the throat of the resonator vibrates, while the elasticity of the air in the cavity of the resonator ensures the necessary restoring force" [4]. In any actual resonator, losses of vibrational energy are inevitable. They can be taken into account in a general form.

The amplifier and its properties will be determined below.

The conditions for the appearance of instability in mechanical models could have been investigated on the basis of the d'Alembert principle, which is an analogue of the second Kirchhoff law in electrical engineering, which is used, for example, in the analysis of the Van der Pol electronic self-excited oscillator [1]. But as experience shows, sometimes this leads to the loss of a substantial portion of information. The solution of the problem on the basis of the energy conservation law in its vibrational form gives the fullest result. As applied to the model considered, the equation corresponding to this approach can be written in the form

$$dW_{\mathrm{k}}/dt + dW_{\mathrm{r}}/dt + dW_{\mathrm{p}}/dt = dW_T/dt. \tag{1}$$

According to [2], the terms on the left-hand side of Eq. (1) can be represented in the form of displacement functions of the kinetic element of the resonator x_m and the corresponding time derivatives:

$$dW_{\mathrm{k}}/dt = m\,(d^2x_m/dt^2)\,(dx_m/dt)\,, \tag{2}$$

$$dW_{\mathrm{r}}/dt = r\,(dx_m/dt)^2\,, \tag{3}$$

$$dW_{\mathrm{p}}/dt = (dx_m/dt)\,x_m C^{-1}\,. \tag{4}$$

To bring Eq. (1) to a form suitable for solution, it is also necessary that its right-hand, thermal, side also be determined in terms of x_m.

According to [5], the rate of the combustion reaction which is described in the context of molecular-kinetic concepts as the mean number of the events of exothermal interactions of the particles of a fuel mixture per unit time in a unit volume, can be represented as a certain function f of the temperature T and pressure p; the expansion of this function into a Taylor series in small changes of the pressure $p_{\sim}$ has the form

$$f = f\,\overline{(p,\,T)} + \overline{(\partial f/\partial p + dT/dp \cdot \partial f/\partial T)}\,p_{\sim} + \dots\,. \tag{5}$$

The expressions under the bars are taken at the moment of the start of reading.

If in each event of exothermal interaction the quantity of heat q is evolved on the average, then for the time t from the start of reading a unit volume of fuel mixture will evolve the quantity of heat

$$Q = \int_0^t f\,q\,dt\,.$$

As a result, the temperature of the gas mixture will increase by

$$\Delta T = Q/c\rho\,.$$

As follows from the equation of the gas state, this will lead to an increase in the pressure in the considered unit volume by

$$\Delta p = kq\,(cM)^{-1} \int_0^t f\,dt\,. \tag{6}$$

Here, it is taken into account that the frequency of the events of heat removal is much greater than the frequency of possible vibrations, the rate of the processes of heat and mass transfer is much smaller than the speed of sound, and the total number of particles after their interaction is taken to be constant.

In principle, self-oscillation is a nonlinear process [1], and the limited nature of the amplitude of the harmonic fluctuations developed is attributed precisely to the nonlinearity of certain parameters and properties of the system. However, the possibility of the solution of this problem as a nonlinear one is virtually excluded today. And not only because of the mathematical difficulties that accompany this solution, but mainly due to the absence of acceptable analytical descriptions of the process of combustion and of its rate f. Therefore, we will investigate the problem in a linear approximation, in the regime of small amplitudes of vibrations, when this approximation remains valid, while the available generalized concepts about the kinetics of the process of combustion (5) turn out to a certain extent to be sufficient for this investigation. Thus, we will elucidate in essence only the qualitative conditions for the excitation of self-oscillations in the so-called "soft" regime, their form and tendencies of changes in the initial period under fixed conditions. The approximation is realized by the linearization of the elastic element of the resonator [4] and by the limitation from above of the series in expression (5) by the term proportional to the first degree of pulsations. As a result of the latter limitation the pressure change (6) will be represented by a sum of two components, one of which increases linearly in time, while the other, which is of interest for the problem considered and which is equal to

$$p_{T\sim} = \Gamma \int_0^t p_\sim\,dt\,, \tag{7}$$

changes in line with the changes in $p_\sim$. Here

$$\Gamma = kq\,(cM)^{-1}\,\overline{(\partial f/\partial p + dT/dp \cdot \partial f/\partial T)}\,.$$

We will call the factor Γ in Eq., (7) the thermoacoustic coefficient.

From Eq. (7) it follows that the pressure $p_{T\sim}$ differs from zero only when the coefficient Γ is not equal to zero, i.e., if the combustion rate depends on pressure and temperature and the liberation of the heat of reaction differs from zero. These conditions are typical for the process of combustion. It is clear here that Γ is larger than zero, since with an increase in both pressure and temperature, the rate of combustion increases. Consequently, $p_{T\sim}$ is associated with the fact of combustion, while the character of relationship (7) allows one to consider a burning fuel mixture as a pressure amplifier $p_\sim$. The latter becomes more wident if $p_\sim$ is presented in a complex form as a component of the Fourier-expansion of the frequency ω:

$$p_{T\sim} = (\Gamma/j\omega)\,p_\sim\,.$$

The proportionality factor $(\Gamma/j\omega)$ is conceptually the amplification factor of the pressure $p_\sim$, which changes harmonically with the frequency ω. As we see, it depends on frequency and can be much greater than unity. The amplified pressure lags in phase relative to that being amplified and exists along with it.

Thus, the third and final element of the self-oscillating system is the amplifier of perturbations, whose functions are fulfilled by the burning fuel mixture. The properties of this amplifier determine the compensation of the losses of vibrational energy in the system considered. Its output power [6] is

$$dW_T/dt = \int_{(V)} (dw/dt)\,dV = -\int_{(V)} \mathrm{div}\,\mathbf{U}\,dV = -\int_{(V)} \mathrm{div}\,(p_{T\sim} \cdot d\mathbf{x}/dt)\,dV\,. \tag{8}$$

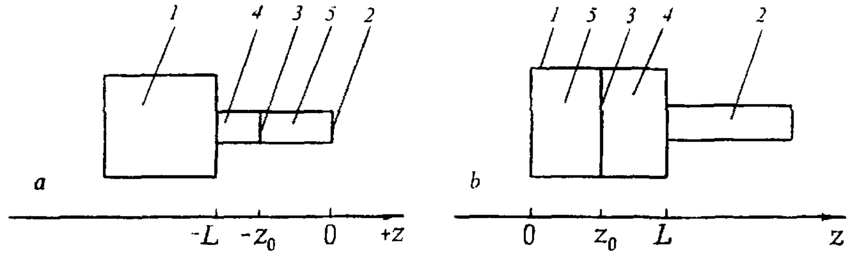

Fig. 1. Model of Helmholtz resonator: a) combustion in throat; b) combustion in cavity.

Thus, in Eq. (1) only the pressure $p_{T\sim}$ remains undefined in terms of displacement, since this procedure depends on localization of the zone of combustion (not yet selected) in the resonator. For this zone two variants are possible:

a) Combustion of the fuel mixture is concentrated in the throat of the Helmholtz resonator; this model is adopted, for example, for radiant tunnel burners [7].

b) Combustion of the fuel mixture is concentrated in the cavity of the Helmholtz resonator. This model was used to investigate low-frequency thermoacoustic instability in liquid-propellant rocket engines (LPRE) and "detonation" in piston carburetor internal combustion engines [8].

Equation (1) on its right-hand side will differ for these variants. Therefore, in what follows it is advisable to carry out analysis on two models distinguished by the position of the zone of heat supply (see Fig. 1). On the schemes of the models, 1 denotes the elastic volume, 2 the throat, 3 the front of the flame, 4 the fuel mixture, and 5 the combustion products.

For both models the positive direction of the coordinate axis is selected in conformity with the direction of the acoustic energy flux.

Frontal Combustion in the Throat of a Helmholtz Resonator. The scheme of the model is presented in Fig. 1a. The flame propagates in the direction of the negative values of the z axis. The mixing chamber plays the role of the elastic volume of the resonator in the tunnel burner modeled, and the gas in the tunnels in which the fuel mixture burns plays the role of the inertial element [7].

Since combustion is concentrated in the throat of the resonator, the distribution of $p_{T\sim}$ will depend on the distribution of $p_\sim$ in it. Taking account of the fact that the geometric dimensions of the resonator are much smaller than the wavelength of possible vibrations, it is possible to assume that the resistance of the emission of the throat and of the mass added to it tend to zero [4]. Then the pressure $p_\sim$ in the throat of the resonator can be presented as a function of the z coordinate:

$$p_\sim = -\rho_0 z\, d^2x_m/dt^2 .$$

Consequently, since the gas in the throat of the resonator moves as a single whole, the pressure $p_\sim$ increases with distance from the throat exit into the resonator. Assuming the integration constant to be equal to zero, it is possible to determine $p_{T\sim}$ from Eq. (7):

$$p_{T\sim} = -\Gamma \rho_0 z\, dx_m/dt . \tag{9}$$

Substituting Eq. (9) into Eq. (8) and then Eqs. (2)-(4) and (8) into Eq. (1) and making obvious transformations, we obtain a second-order differential equation for x_m:

$$d^2x_m/dt^2 + 2(\delta - \chi)\, dx_m/dt + \omega_0^2 x_m = 0 ,$$

where $\delta = r/2m$; $\chi = \Gamma\rho_0 z/2\rho L|_{-\Lambda}^{0}$; $\omega_0^2 = (mC)^{-1}$; 0 and L are the limits of integration over z. Its solution has the form

$$x_m = x_0 \exp(\alpha t), \tag{10}$$

where x_0 is the initial perturbation, and

$$\alpha = -(\delta - \chi) \pm [(\delta - \chi)^2 - \omega_0^2]^{1/2}. \tag{11}$$

From Eq. (11) it is seen that when $(\delta - \chi)^2 < \omega_0^2$, solution (10) describes the pseudoharmonic vibrational motion of the gas in the throat with frequency

$$\nu = [-(\delta - \chi)^2 + \omega_0^2]^{1/2},$$

which is smaller than the natural frequency of the resonator ω_0. When $\delta < \chi$, the amplitude of vibrations increases, which must be interpreted as the development of instability. Otherwise the vibrations are suppressed, and this is indicative of the stability of the system.

We note not an obvious but important result: if within the region of integration Γ changes continuously from zero in the original fuel mixture through a certain maximum value to zero in the combustion products, the coefficient χ under any other conditions is equal to zero and, consequently, the system is stable. A necessary, albeit insufficient, condition for instability is discontinuity of the parameters that determine Γ over the coordinate of change. This discontinuity appears, for example, when turbulization of the flame front occurs. The result described in [7] corresponds to this result, though it is interpreted otherwise by the authors of the work. We are not aware of cases of instability in laminar combustion.

When the front of the flame is turbulent, there is no combustion in the original fuel mixture and, consequently, $\Gamma = 0$, whereas in the section $z = z_0$ the mixture ignites, as a result of which Γ increases by a jump up to the value Γ_0. Then, changing continuously in the process of the burning of the mixture, Γ falls to zero following an arbitrary law. Under these conditions

$$\chi = \Gamma_0 \rho_0 z_0 / 2\rho L.$$

Thus, if δ is a certain inherent characteristic of the resonator, then the value of χ is controllable: with displacement of the turbulent surface of the flame front toward the exit from the throat of the resonator, z_0/L decreases, other conditions being equal, and this leads to a decrease in χ, i.e., to an increase in the stability of the system. This is also confirmed experimentally [7]. It is important that the value of χ be independent of the character of change in Γ over the segments of continuity, but it is determined only by the value of the jump in Γ on the discontinuity surface.

Conditions are possible under which $(\delta - \chi)^2 > \omega_0^2$. If in this case $\chi > \delta$, solution (10) describes an anharmonic relaxational process which acquires the character of an explosion. Presumably, precisely this can explain the "puffs" that often accompany the process of combustion in the limited volume of a combustion chamber.

Frontal Combustion in Cavity of Helmholtz Resonator. The scheme of the model is presented in Fig. 1b. The flame propagates in the direction of positive values of the z axis. The role of the elastic volume in both piston and liquid-propellant rocket engines is played by the combustion chamber. The piston in a piston engine is an inertial element; its mobility is limited by the clearance in the fit of its connection with a crank by means of a journal [8]. In an LPRE, the inertial element is the mass of the fuel in the supply mains, which, as is known [9], in the case of low-frequency instability performs vibrational motion in them, which is superimposed on a constant flux. For the time being we neglect acoustic losses through the nozzle of the rocket engine, assuming them to be small in comparison with the losses in the fuel mains.

We assume that the cross sectional area of the cavity of the resonator in any plane perpendicular to the ordinate axis is constant. We also note that in the case of this model the inertial element of the resonator for specific devices can be of any form: gaseous, liquid or solid. Only its mass is important for calculation.

The change in comparison with the previous model for the position of the front of the flame in the resonator (in its cavity rather than in the throat) changes the representation of the power dW_T/dt. We will consider this.

The dynamic pressure at all points of the elastic volume of the Helmholtz resonator changes identically. But the vibrational velocity and displacement are not identical; they depend on position. Since [4]

$$p_{\sim} = x_m/CS_m = x_z/C_zS\,, \tag{12}$$

then

$$dx_z/dt = (dx_m/dt)\, C_zS/CS_m\,, \tag{13}$$

where $C_z = z/\gamma PS$ is the elasticity of the portion in the volume of the cavity to the left of the cross section with the z coordinate; $C = SL/\gamma PS_m^2$. After the substitution of Eqs. (7), (12), and (13) into Eq. (8) we obtain

$$dW_T/dt = -\,(dx_m/dt)\,\Gamma z\,(CL)^{-1} \int_0^t x_m\, dt\Big|_0^L\,. \tag{14}$$

The substitution of Eqs. (2)-(4) and (14) into Eq. (1) and transformations lead to a third-order differential equation for x_m:

$$d^3x_m/dt^3 + \delta d^2x_m/dt^2 + \omega_0^2\, dx_m/dt + \omega_0^2\, \psi x_m = 0\,, \tag{15}$$

where $\delta = r/m$; $\omega_0^2 = (mC)^{-1}$; $\psi = \Gamma z/L|_0^{\Lambda}$.

The solution of Eq. (15) for the initial perturbation x_0 has the form

$$x_m = x_0\,[\exp(\alpha t) + 2\exp(\beta t)\cos \nu t]\,, \tag{16}$$

where α, β, and ν are determined rather cumbersomely in terms of the coefficients of Eq. (15) in conformity with the Cardan solution [10] of the cubical characteristic equation.

Analysis shows that α is always smaller than zero, and the first term in Eq. (16) tends to zero with time. The second term describes quasiperiodic vibrations with frequency ν whose amplitude decreases or increases depending on the sign of β. For a high-Q resonator it is possible to assume that $\delta \ll \omega_0$. Under this condition the expressions for β and ν become more obvious:

$$\beta \approx (\psi - \delta)/2\,, \tag{17}$$

$$\nu \approx \omega_0 \left\{1 + [(3\psi - \delta)/2\omega_0]^2/2\right\}. \tag{18}$$

From Eq. (16), with account for Eq. (17), it follows that the stability of the system depends on the relationship between ψ and δ. If $\psi > \delta$, the exponent β is larger than zero and, according to Eq. (16), a small perturbation x_0 causes increasing vibrations with frequency ν in the system, i.e., a self-oscillating process develops. Otherwise, the vibrations decay.

Similarly to the result of the analysis of the first model, if Γ within the range of integration changes continuously, the system is stable. If the value of Γ in the section z_0 increases in a jumpwise fashion by the value Γ_0, the coefficient ψ can be defined as

$$\psi = \Gamma_0\, z_0/L\,.$$

Consequently, the instability of the system increases as the flame front approaches $z = L$, i.e., the piston or the injector head. This dependence shows up in investigations in both carburetor [11] ("detonation") and rocket [9] (low-frequency instability) engines.

Of interest is the result that follows from Eq. (18): in contrast to the system that is described by a second-order differential equation, the frequency of vibrations appearing in a system described by a third-order differential equation is always higher than the frequency of its natural vibrations ω_0. This character of the effect of the process of combustion on the frequency of vibrations in this model is known from experiments with carburetor engines [12], but it has not been described analytically earlier.

Probably, a similar result is also observed in an LPRE: when a diergolic nitric acid–octane fuel pair is replaced by a self-igniting nitric acid–furfuryl alcohol pair, which must be characterized by a higher value of Γ_0 due to its higher reactivity, the frequency of pulsations increases by a factor of 2–3, other conditions being equal [9]. The hypothetical character of the comparison in this case is explained by the fact that the coefficient Γ, which reflects real and important properties of fuel mixtures, has never been the subject of either a theoretical or experimental investigation, and its value for any fuel is not yet known.

We will note some of the consequences that are common for the two models considered.

The feedback in the system finds its reflection in expression (7): an increase in pressure $p_{\sim}$ leads to an increase in $p_{T\sim}$ which, in turn, increases the rate of combustion, the resulting pressure, and so on. Consequently, due to this feedback a self-sustaining process of perturbation increase occurs that develops in the considered models with specific features that are attributed, as follows from the previous description, to the fact that it occurs in a resonator.

It is possible also to follow the influence of the change in the sign of feedback on the system. The direction of the front of the flame selected in the problems considered with respect to the direction of the acoustic energy flux leads to satisfaction of the instability condition. This suggests a positive nature of feedback. By reversing the direction of the front or of the energy flux, the sign of the right-hand side of Eq. (1) will change, the feedback will become negative, and the system will be stable. In particular, low-frequency instability does not appear in an LPRE if the losses through the nozzle are larger than losses in the fuel mains. This is achieved by an increase in the area ratio of the nozzle, which not only increases losses, but also changes the direction of the acoustic energy flux.

Thus, in application to the model of unstable combustion in a resonator with lumped parameters, all the characteristic elements of self-oscillating systems that were listed in the introductory part of the article were isolated and considered in their interrelation, and relations were obtained that determine the conditions of instability for two possible positions of the zone of heat removal in a resonator. Today these relations do not allow quantitative evaluations of the stability of combustion in devices that can be modeled by a Helmholtz resonator, since due to the absence of any other, they are based on a nonquantitative representation of the process of combustion (5). But they allow one to evaluate qualitatively the effect on stability of the factors that are taken into account theoretically, to compare this effect with that observed in laboratory experiments and in actual technical devices, and to purposefully affect the process. The validity of this state follows from the fact that experimental results presented, for example, in [7, 9, 11-13], and in a multitude of other works agree qualitatively well with theoretical results, though sometimes they were obtained without fixing all the other conditions that are necessary for a rigorous evaluation.

A large number of the results of long-standing experimental investigations of "detonation" of motor fuel in gasoline engines obtained by many authors "have averaged" the scatter in the results caused by the errors of complex experiments to the extent that it was possible to represent them as facts and dependences that do not cause any doubts. A comparison of scores of these dependences and facts with the results of the analysis carried out above is given in [14]. It showed their good agreement with one another, despite the fact that the well-known Arrhenius-type equation [15] used in this case to describe the mechanics of burning corresponds more or less satisfactorily (in the very first approximation and only with account for the results of its analysis carried out by Ya. B. Zel'dovich [13]) to the actual exothermal process, to the process of combustion.

The fact that in all of the actual technical devices enumerated above the design solutions used led spontaneously to the undesired satisfaction of the conditions of instability can be considered as a "technical mishap." Its consequences can be eliminated in principle on the basis of the concepts stated above.

NOTATION

z, coordinate axis; z_0, coordinate of discontinuity surface of parameters; x, acoustic displacement in an arbitrary section z; x_m, acoustic displacement of inertial element of resonator; m, mass of inertial element of resonator; C, flexibility of cavity of the resonator; V, total volume of resonator; r, generalized loss resistance; C_z, flexibility of portion of cavity of resonator in inlet section z; $d\mathbf{x}/dt$, velocity vector in section z; W_k, W_p, W_r, kinetic and potential energies and loss energy in resonator; W_T, vibrational compound of energy generated by zone of heat supply; w, volumetric density of acoustic energy; S, S_m, cross sectional area of cavity and throat of resonator; L, coordinate; T and ΔT, temperature and increment in temperature; $p_{\sim}$, perturbation of pressure; $p_{T_{\sim}}$, pressure increment (increased pressure); P, static pressure; Q, quantity of heat; c, specific heat; k, Boltzmann constant; M, mean mass of particles in medium; q, heat output of single event of reaction; j, imaginary unit; t, time; ω, frequency; ρ, density of fuel mixture; ρ_0, density of combustion products; f, rate of reaction of combustion; γ, ratio of specific heats; δ, χ, ψ, ω_0, coefficients of differential equations; ν, frequency of self-oscillations; Γ, thermoacoustic coefficient; Γ_0, jump in value of Γ at section z_0; U, Umov vector.

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